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On the Moduli Dependence of Nonperturbative Superpotentials in Brane Inflation

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Abstract

We discuss string corrections to the effective potential in various models of brane inflation. These corrections contribute to the mass of the inflaton candidate and may improve its slow-roll properties. In particular, in orientifold string compactifications with dynamical D3- and D7-branes, the corrections induce inflaton dependence in the part of the superpotential that arises from gaugino condensation or other nonperturbative effects. The additional terms are in part required by supersymmetry. We explicitly discuss D3/D7-inflation, where flat directions of the potential can be lifted, and the KKLMMT model of warped brane inflation, in which the corrections open up the possibility of flattening the potential and canceling unwanted contributions to the inflaton mass.

1 The inflaton mass problem and the rho problem

While cosmological inflation is a very attractive mechanism to solve many of the inherent problems of standard big bang cosmology, there are still many open questions about how it can be realized within string theory. Let us single out two obstacles that played an important role in the development of the models considered below. Firstly, the no-go theorems of [1] show that it is not possible to achieve a compactification of ten- or eleven-dimensional supergravity with a positive cosmological constant in four dimensions, at least at leading order in an expansion in derivatives and in the string coupling. This conclusion is not changed even by including orientifold planes, non-dynamical objects of formally negative energy density, in the background [2]. The way out of this problem, which in its present concrete form was proposed by Kachru, Kallosh, Linde and Trivedi [3], is to add nonperturbative effects of some sort. The resulting effective superpotential is our main concern in this note. The second obstacle, the "inflaton mass problem", sometimes also called the "eta problem", can be seen in terms of purely four-dimensional $\mathcal{N}=1$ effective supergravity, which says that the standard form of the F-term potential,

$$\mathcal{V}^F = e^{\mathcal{K}}(\mathcal{K}^{I\bar{J}} D_I W D_{\bar{J}}\bar{W} - 3|W|^2) = e^{\mathcal{K}}\tilde{\mathcal{V}}^F \ , \tag{1}$$

given an expansion of the Kähler potential $\mathcal{K} = \phi\bar{\phi} + \cdots$ in the fluctuations of the inflaton around $\langle\phi\rangle = 0$, produces

$$\frac{\mathcal{V}^F_{\phi\bar{\phi}}}{\mathcal{V}^F} = 1 + \frac{\tilde{\mathcal{V}}^F_{\phi\bar{\phi}}}{\tilde{\mathcal{V}}^F} \quad \Rightarrow \quad \eta \sim \mathcal{O}(1) \ , \tag{2}$$

i.e. a contribution of order one to the slow roll parameter η [4].[1] Obviously, this can be avoided by invoking a cancellation between the two terms in (2), moderately fine-tuning (at the level of 1%) to achieve slow-roll at a comfortable $\eta \sim 0.01$. Still, this means slow-roll inflation can be realized but is not a generic property. One might then ask, given the fact that a supergravity potential generated by spontaneous supersymmetry breaking has an inflaton mass problem, could one use explicit supersymmetry breaking within string theory, e.g. anti-branes in otherwise supersymmetric backgrounds [5]? Then the brane moduli, such as their positions, are interpreted as the scalar inflaton fields [6]. Unfortunately, the simplest version of the brane/anti-brane scenario (in an unwarped background) produces a classical Coulomb brane/anti-brane attraction that

[1]Since the other slow-roll parameter ϵ is usually much smaller than η in the models we consider, it will be ignored in the following.

is too strong to allow for slow-rolling, again $\eta \sim \mathcal{O}(1)$, and the inflation mass problem from supergravity recurs.

There are, in fact, various ways to get around the inflaton mass problem within brane inflation. We will focus on two approaches here: One is the setting of D3/D7-brane inflation [7, 8, 9, 10], which starts from an orientifold compactification with D3- and D7-branes and $\mathcal{N} = 2$ supersymmetry, then breaks supersymmetry completely by a small brane misalignment. This is described by an effective Fayet-Iliopoulos term (or better, a triplet of such terms). Thus the scenario is similar to D-term inflation [11] and therefore escapes the inflaton mass problem.

The other approach is the model of Kachru, Kallosh, Linde, Maldacena, McAllister and Trivedi [12], who suggested that the brane/anti-brane Coulomb attraction can be weakened through gravitational redshift, if the background metric includes a warp factor which differs at the position of the brane and the anti-brane by a sufficiently large amount. The KKLMMT model contains the following ingredients:

1. The model is an orientifold compactification of type IIB on a (warped) Calabi-Yau with background RR and NSNS 3-form fluxes [2].
2. The warping is such that there are regions with very strong warping, usually modeled by the Klebanov-Strassler geometry [13].
3. As a prototype of a nonperturbative effect, one considers strongly coupled gauge dynamics on D7-branes wrapped on 4-cycles of the Calabi-Yau.
4. The inflaton scalar field arises as the position field of dynamical D3-branes.
5. Supersymmetry is broken either through anti-D3-branes [12] or through non-BPS world-volume gauge field backgrounds on the D7-branes [14]. In the case of anti-D3-branes, it is assumed that the gravitational redshift is big enough to allow neglecting the Coulomb interaction in the potential.

The models have many similarities, but also differ in important ways. In particular, points 2 and 3 of our list do not play any major role in the model of [7], as the relevant brane interaction in that model occurs within a single D3/D7-system (cf. fig. 4 of [7]).[2] This requires a sufficient separation of the single D3/D7-system from the rest of the D7-branes. More generally, strong gauge dynamics on these D7-branes will also play a

[2]We thank R. Kallosh for pointing this out to us.

role in $\mathcal{N} = 2$ models of D3/D7-inflation and the results below are in general equally relevant for that case.

The potential of the KKLMMT model is assembled from the following contributions. The action for the bulk Calabi-Yau was derived through generalized dimensional reduction and related to gauged supergravity (see e.g. [15, 16]). The RR and NSNS 3-form fluxes add a superpotential of the form [17]

$$W_{3-\text{flux}} = \int_{\text{CY}} \Omega_3 \wedge (F_3 - \tau H_3) \ , \tag{3}$$

which depends on the complex dilaton τ and the complex structure moduli u^I. In general, the supersymmetry conditions $D_\tau W = D_{u^I} W = 0$ can be strong enough to fix all these fields. It is then assumed that their masses upon stabilization become large enough that τ and u^I can be completely ignored during inflation (also, the string coupling should be fixed in the perturbative regime). Furthermore, as was discussed in [18], one can argue that the open string moduli of D7-branes are fixed at the same mass scale. Then, the only relevant degrees of freedom below this scale are the Kähler moduli and the D3-brane scalars, and in the simple case of only one Kähler modulus ρ, one may restrict to a model of only two fields: the Kähler modulus ρ and the inflaton ϕ. Gaugino condensation on the D7-branes induces a nonperturbative superpotential involving the gauge kinetic function f_7 of the D7-brane world volume gauge theory,

$$W \ = \ W_{3-\text{flux}} + W_{\text{nonpert}} \ \sim \ W_0 + Ce^{-af_7} \tag{4}$$

for some constants a, C and W_0, where W_0 is the value of the flux-induced superpotential at the minimum of the potential.[3] A priori, f_7 is a holomorphic function of all the chiral fields, $f_7 = f_7(\rho, \phi)$. Now one can find supersymmetric vacua with negative cosmological constant by imposing $D_\rho W = 0$, with stabilized (and moderately large) volume. The Kähler potential for this ρ-ϕ model has been conjectured to be of the general form [16]

$$\mathcal{K} = -3 \ln[-i(\rho - \bar\rho) + k(\phi, \bar\phi)] \tag{5}$$

to leading order, where $k(\phi, \bar\phi)$ is the Kähler potential of the Calabi-Yau manifold. For small ϕ it can be approximated as $k(\phi, \bar\phi) = \phi\bar\phi + \cdots$, and we ignore the higher terms in what follows. It is important that the argument of the logarithm is still equal to the Calabi-Yau volume in the Einstein frame even when $\phi \neq 0$, i.e.

$$-\, i(\rho - \bar\rho) + \phi\bar\phi \ = \ e^{-\Phi} v^{2/3} \ , \tag{6}$$

[3] The notation in (4) is not meant to imply that W_{nonpert} cannot also depend on the 3-form fluxes. The gauge kinetic function f_7 may receive corrections in their presence.

where v is the volume in the string frame. Eq. (6) implies a mixing of geometrical closed string moduli (v) and open string moduli (ϕ) in the Kähler coordinate ρ. This mixing gives rise to two problems: the "rho problem" and the inflaton mass problem, which we shall return to in a moment. Finally, the last contribution to the potential that was added in [12] is the one due to anti-D3-branes (that are non-dynamical in the presence of imaginary self-dual 3-form flux [19]) which is written as a "warped FI-term":

$$e^{-\Phi}\sqrt{-\mathrm{P}[g_4]} \quad \xrightarrow{\text{Einstein frame}} \quad \mathcal{V}^D = \frac{D}{[-i(\rho-\bar{\rho})+\phi\bar{\phi}]^2} \ , \tag{7}$$

where D is a constant given by the 3-form fluxes and the tension of the anti-branes [12]. This can be derived by reducing the Born-Infeld action in the warped background, the constant D being the warp factor at their position, at the tip of the warped throat. If we ignore the dependence of f_7 on ϕ, i.e. if we set $f_7 = f_7(\rho)$, we have an inflaton mass problem in the model. Indeed, the full F-term potential reads (with $W_\rho = \partial_\rho W$, etc.)

$$\begin{aligned} \mathcal{V}^F \ &= \ \frac{1}{3[-i(\rho-\bar{\rho})+\phi\bar{\phi}]^2}\Big(-i(\rho-\bar{\rho})|W_\rho|^2 \\ &\qquad -3i(W_\rho\bar{W}-\text{c.c.}) - |W_\phi|^2 - i(\phi W_\phi \bar{W}_{\bar{\rho}} - \text{c.c.})\Big) \ . \end{aligned} \tag{8}$$

Then, for $f_7 = f_7(\rho)$ or $W = W(\rho)$, the potential $\mathcal{V} = \mathcal{V}^F + \mathcal{V}^D$ depends on ϕ only through the prefactor $[-i(\rho-\bar{\rho})+\phi\bar{\phi}]^{-2}$ and it follows[4]

$$\mathcal{V}_{\phi\bar{\phi}} \ \propto \ \mathcal{V} \quad \Rightarrow \quad \eta \sim \mathcal{O}(1) \ . \tag{9}$$

There is nothing to tune, so the inflaton mass problem looks incurable at this level. It was also argued that this mass term is related to the conformal coupling on the 3-brane world-volume and thus (9) was to be expected [12, 20]. Fortunately, there are contributions to η that we have neglected until now, and they may be useful for canceling the above terms. Indeed, f_7 may depend on ϕ :

$$f_7 = f_7(\rho,\phi) \quad \Rightarrow \quad W_{\text{nonpert}} \sim e^{-af_7} = w(\phi)e^{ia\rho} \ . \tag{10}$$

Computing the function $w(\phi)$ in a controllable, sufficiently simple model was the goal of [21].[5]

[4]Note that $\eta \sim \mathcal{O}(1)$ becomes obvious only after normalizing the inflaton field correctly, i.e. after introducing the canonically normalized inflaton $\varphi \sim \phi/\sqrt{-i(\rho-\bar{\rho})}$ [12].

[5]There have been alternative approaches, such as [22], where a cancellation among the above mass contribution and the Coulomb interaction was designed. However, such a cancellation requires rather small warping, because the Coulomb interaction between the D3-branes and anti-D3-branes is suppressed by the warp factor. Thus it is a setup slightly different from that considered here.

To understand how the dependence of f_7 on ϕ arises, it is very useful to consider the following puzzle, the "rho problem", and its resolution. The gauge coupling constant on the D7-branes is at leading order given by the dimensional reduction of the Born-Infeld action through

$$e^{-\Phi}\mathrm{tr}\sqrt{-\det(\mathrm{P}[G]+\mathcal{F}_7)} \quad \sim \quad \underbrace{e^{-\Phi}v^{2/3}}_{=-i(\rho-\bar{\rho})+\phi\bar{\phi}}\sqrt{-g_4}\ \mathrm{tr}\,\mathcal{F}_7^2\ . \tag{11}$$

By supersymmetry, the gauge coupling must be the real part of a holomorphic function, but it seems that it is not! Clearly, $-i(\rho-\bar{\rho})+\phi\bar{\phi}$ is not the real part of any function holomorphic in ρ and ϕ. Thus, it looks as if we are facing a violation of supersymmetry by violation of holomorphy [8]. Two observations hint towards the solution: *i*) For a stack of D3-branes, the field ϕ transforms in some representation of the D3-brane gauge group, and thus $\phi\bar{\phi}$ should really be read as $\phi^a\bar{\phi}^a = \mathrm{tr}\,\phi\bar{\phi}$. Then, the term $\mathrm{tr}\ \phi\bar{\phi}\,\mathrm{tr}\,\mathcal{F}_7^2$ carries two traces, and has to come from a string diagram with at least two boundaries. *ii*) At $\phi=0$, the volume modulus ρ contains the factor $e^{-\Phi}$ whereas $\mathrm{tr}\ \phi\bar{\phi}$ does not, and so the two terms arise at different orders of string perturbation theory; one from the disk, the other from open string one-loop level (annulus plus Möbius strip). In all, the suspicion arises that there could be a one-loop contribution from an annulus diagram with two boundaries, one on the D7- the other on the D3-branes, which may produce just the right dependence of the D7-brane gauge coupling to reinstate holomorphy and hence supersymmetry, cf. fig. 1.

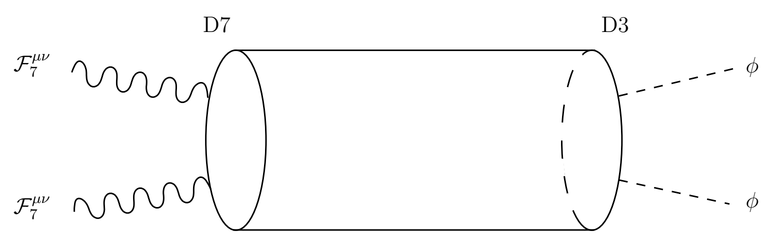

FIGURE 1: The 3-7 annulus

This argument turns out to be correct [21]. In fact, the rho problem, that we only invoked as a clue towards addressing the inflaton mass problem, is more general than the application in the cosmological model at hand, and arises in any compactification that includes mobile D-branes. In addition, the same 3-7 annulus diagram leads to the above mentioned dependence of the superpotential on the inflaton field ϕ after gaugino condensation.

We will not repeat any details of the computations of [21], only discuss and interpret the results. We do add one ingredient relative to [21]: an alternative and independent calculation of the relevant terms using standard string perturbation theory instead of the background field formalism. This provides an independent confirmation of our earlier results and is an independent check of the validity of the background field method in the problem at hand. The calculation is given in appendix A.

2 On D3/D7-brane inflation

In this section we comment on the simplest setting in which the corrections to the gauge kinetic functions of D7-branes play a role, the model of D3/D7-brane inflation. As this model has more moduli than ρ and ϕ, it will not be sufficient to use the $\mathcal{N}=1$ notation of the previous section, and we need more general expressions for the effective Lagrangian in this section only. In the next section on warped brane inflation with $\mathcal{N}=1$ supersymmetry, we shall switch back to ρ and ϕ, essentially by truncating the $\mathcal{N}=2$ model.

In order to explicitly calculate the relevant one-loop diagrams shown in figure 1, we use the simplest available orientifold model with D3- and D7-branes: the $\mathcal{N}=2$ type IIB orientifold on $\mathbb{T}^2 \times \mathrm{K3} = \mathbb{T}^2 \times \mathbb{T}^4/\mathbb{Z}_2$, where the $\mathbb{Z}_2$ is the reflection along the four circles [23, 24], and the orientifold projection is the T-dual of the world sheet parity Ω, i.e. $\Omega' = \Omega R^6 (-1)^{F_L}$, where R^6 is the reflection along all six coordinates.[6] This is precisely the model discussed in the context of $\mathcal{N}=2$ D3/D7-inflation [7], before adding the deformation that breaks supersymmetry. The $\mathcal{N}=2$ gauged supergravity Lagrangian of this compactification, including the 3-form RR and NSNS fluxes, was discussed in depth in [25, 26]. The model has 32 D3-branes and 32 D7-branes with maximal gauge group $U(16)_9 \times U(16)_5$, the D7-branes wrapping the K3. Here we are only interested in the vector multiplet sector of the full theory, but including the branes and their open string degrees of freedom. In total, there are three vector multiplets from the KK reduction of the bulk fields and the vector multiplets that arise from the D3- and D7-branes, which in general carry a representation of their non-abelian gauge groups. In the abelian limit, where this group is broken through an adjoint Higgs

[6] The results of [21] were mostly given in the T-dual picture with D9/D5-branes, but are unaffected by this T-duality, up to the standard transformation of the fields. In particular, all amplitudes computed there for D9/D5-branes are identical for D3/D7-branes after exchanging momentum and winding modes. Our alternative calculation in the appendix is performed directly in D3/D7-brane language, and there we also check the T-duality mapping.

mechanism, these can be replaced by 16 + 16 abelian vector multiplets. The scalars in the bulk multiplets will be denoted S, S' and U, and the open string scalars A_i for D3-branes and A_a for D7-branes. The latter are defined in terms of the geometrical positions of the stacks of D3- and D7-branes on the $\mathbb{T}^2$, which are the real two-vectors $(a^4, a^5)_i$ for D3-brane scalars and $(a^4, a^5)_a$ for D7-brane scalars, and the position vectors are conveniently complexified to $A = a^4 + Ua^5$, for both types of indices.[7]

In the absence of open string scalars (i.e. setting $A = 0$) the three scalars in the three bulk vector multiplets are defined as[8]

$$S|_{A=0} = \frac{1}{2\pi\sqrt{2}} \left(C_{(0)} + ie^{-\Phi_{10}} \right) \ , \quad S'|_{A=0} = \frac{1}{2\pi\sqrt{2}} \left(C_{(4)} + ie^{-\Phi_{10}} \mathrm{vol(K3)} \alpha'^{-2} \right) \ , \tag{12}$$

where the $C_{(p)}$ are the scalars arising from the RR p-forms, and $U|_{A=0} = (G_{45} + i\sqrt{G})/G_{44}$ for the complex structure of the $\mathbb{T}^2$. It was shown in [27] that dimensional reduction of type I supergravity in the absence of open string moduli is described by the prepotential $\mathcal{F}|_{A=0} = SS'U|_{A=0}$. In this limit, the gauge coupling of the D7-branes is given by the real part of

$$f_7|_{A=0} = -iS'|_{A=0} \ , \tag{13}$$

which follows from the Born-Infeld action. In the presence of the open string vector multiplets, the fields S, S' have to be modified via

$$S = S|_{A=0} + \frac{1}{8\pi} \sum_a (a^5)_a A_a \ , \quad S' = S'|_{A=0} + \frac{1}{8\pi} \sum_i (a^5)_i A_i \ , \quad U = U|_{A=0} \ . \tag{14}$$

Dimensional reduction (including certain counterterms) allows one to deduce the corrected Kähler potential [27]

$$\begin{aligned} \mathcal{K} \;\; = \;\; & -\ln \Big[(S - \bar{S})(S' - \bar{S}')(U - \bar{U}) \\ & -\frac{1}{8\pi}(S - \bar{S}) \sum_i (A_i - \bar{A}_i)^2 - \frac{1}{8\pi}(S' - \bar{S}') \sum_a (A_a - \bar{A}_a)^2 \Big] \ , \end{aligned} \tag{15}$$

which follows from a prepotential

$$\mathcal{F} = SS'U - \frac{1}{8\pi} S \sum_i A_i^2 - \frac{1}{8\pi} S' \sum_a A_a^2 \ . \tag{16}$$

[7]Note that these are the proper special-geometry coordinates, T-dual to the ones used in [21].

[8]The conventions of [21] differ from those of e.g. [27], as explained in [21].

The gauge coupling of the D7-brane gauge group is part of the period matrix of the $\mathcal{N} = 2$ Lagrangian defined through this prepotential. To introduce some more compact notation, let us collect all fields in projective coordinates

$$\left(X^\Lambda/X^0\right) = (1, S, S', U, A_i, A_a) \ . \tag{17}$$

with indices

$$\Lambda \in \{0, \dots, 3 + n_7 + n_3\} \ , \quad i \in \{1, \dots, n_3\} \ , \quad a \in \{1, \dots, n_7\} \ . \tag{18}$$

For simplicity, we now assume that there is only a single stack of D3- or D7-branes: $n_3 = 1$ and $n_7 = 1$, hence $\Lambda \in \{0, \dots, 5\}$. Then the D7-brane coupling, as derived from supergravity, is the imaginary part of the 55 entry of the period matrix, denoted $\mathcal{N}_{55}$. Since we are interested in nonperturbative effects on these D7-branes, we focus on the situation that the gauge group is non-abelian. The non-abelian coupling reads

$$\mathcal{N}_{55} = S' \ . \tag{19}$$

We therefore recover the $\mathcal{N} = 2$ version of the rho problem, since the correction present in (19) through the corrected definition of S' in (14) does not follow from the Born-Infeld (plus Chern-Simons) action, which only produces $S'|_{A=0}$ as in (13) above (cf. [8, 28]). It follows that dimensional reduction of the tree-level effective action does not produce the correct supersymmetric Lagrangian. We will see that the one-loop annulus correction precisely remedies this discrepancy. But first, we have to discuss another subtlety, in order to make contact with the D3/D7-inflation model of [7, 8, 9], and the $\mathcal{N} = 2$ special geometry Lagrangian of [25, 26, 27].

One of the main features of the Lagrangian considered in [7, 8, 9, 25, 26], meant to describe the string compactification on K3$\times\mathbb{T}^2$ with fluxes, is the symplectic transformation of the coordinate fields X^Λ into a frame where no prepotential exists. In this symplectic section, the Lagrangian was shown to possess a shift symmetry with respect to shifting the real parts $A_i + \bar{A}_i$ of the D3-brane coordinates. Therefore, these fields appear as exactly flat directions of the classical potential, and are candidates for inflaton fields, provided "exactly flat" becomes "nearly flat" through some source of symmetry breaking. Now, our corrections to the D7-brane gauge coupling appear to be computed in the frame where the prepotential (16) exists, so we should apply the symplectic transformation of [25] before any comparison could be made. However, it turns out that this transformation leaves the gauge coupling of the (non-abelian) D7-brane gauge group unchanged, as we now argue.

Let us consider the simplest setting and take the D7-brane gauge group to be unbroken, i.e. all D7-branes in a single stack, and a potentially anomalous overall

$U(1)$ decoupled. Then, the period matrix following from the prepotential (16) does not exhibit any terms coupling the D7-brane gauge fields to any of the other gauge fields. In other words, we have $\mathcal{N}_{5\Lambda} = 0$ for $\Lambda \neq 5$. We now apply the symplectic transformation [29]

$$\mathcal{N} \longrightarrow \mathcal{N}' = (B + A\mathcal{N})(A - B\mathcal{N})^{-1} , \tag{20}$$

with the matrices A and B given in [25]. The 7-brane gauge coupling after the transformation is the imaginary part of

$$\mathcal{N}'_{55} = \sum_{\Lambda}(B + A\mathcal{N})_{5\Lambda}(A - B\mathcal{N})^{-1}_{\Lambda 5} . \tag{21}$$

Due to the simple structure of the matrices A and B we have

$$(B + A\mathcal{N})_{5\Lambda} = \mathcal{N}_{5\Lambda} = \mathcal{N}_{55}\delta_{5\Lambda} , \tag{22}$$

where in the last equality we have used the fact that the 7-brane gauge group is non-abelian. Thus we arrive at

$$\mathcal{N}'_{55} = \mathcal{N}_{55}(A - B\mathcal{N})^{-1}_{55} = \mathcal{N}_{55}\frac{1}{\det(A - B\mathcal{N})} \det|A - B\mathcal{N}|_{55} , \tag{23}$$

where $|A - B\mathcal{N}|_{55}$ denotes the minor M_{55} of the matrix $A - B\mathcal{N}$. Because of $(A - B\mathcal{N})_{5\Lambda} = \delta_{5\Lambda}$, we can use the expansion formula for determinants to see that $\det(A - B\mathcal{N}) = \det|A - B\mathcal{N}|_{55}$ and thus the announced result follows:[9]

$$\mathcal{N}'_{55} = \mathcal{N}_{55} . \tag{25}$$

This shows that the non-abelian D7-brane gauge coupling is unaffected by the symplectic transformation, which justifies simply adding our correction to the entry $\mathcal{N}_{55}$ of the period matrix in the Lagrangian of [8, 9, 26]. We can now put things together to solve the rho problem, and show how the one-loop corrections to the gauge coupling constants break the shift symmetry of the $\mathcal{N} = 2$ Lagrangian of [8, 9, 26, 30].[10] As

[9] Note that for this derivation one only needs that

$$\mathcal{N}_{5\Lambda} = \mathcal{N}_{55}\delta_{5\Lambda} , \ A_{5\Lambda} = \delta_{5\Lambda} , \ B_{5\Lambda} = 0 , \tag{24}$$

so that it does not depend on the details of the matrices A, B used in [25] but holds for every symplectic transformation not mixing the non-abelian 7-brane gauge fields with any of the abelian vectors.

[10] We do not want to imply that this is a problem for these papers. To construct a viable model for inflation, one always has to break the shift symmetry at some point. Our one-loop corrections present one way to do so. An alternative way has been put forward in [31], which makes use of the misalignment of the D3/D7-branes along the lines of [32]. We thank R. Kallosh for discussions on this point.

outlined above, the dependence of the gauge couplings on the open string scalars arises from annulus and Möbius strip string diagrams, i.e. one-loop threshold corrections [33]. The most general expression for the D7-brane gauge coupling constant can be read off from formula (44) in [21], valid for all A_i, A_a distinct and non-vanishing. For the present purpose, it will be sufficient to consider the situation where all D7-branes are located at the origin, i.e. $A_a = 0$,[11] which leads to

$$f_7 = -iS' - \frac{1}{8\pi^2}\sum_i \ln \vartheta_1(A_i, U) + \frac{1}{\pi^2} \ln \eta(U) \ . \tag{26}$$

Note that the one-loop corrections contain a term that completes S' to the modified form (14), thus solving the rho problem in the $\mathcal{N} = 2$ case. Typically, strong coupling effects in the D7-brane gauge group (for instance instantons) then lead to a dependence of the potential on $e^{-\mathrm{Re}\, f_7}$. In this way, the additional holomorphic terms in (26) induce explicit dependence on the real parts $A_i + \bar{A}_i$ of the D3-brane scalars in the potential, and thus lift the shift symmetry of the effective Lagrangian.

3 On warped brane-inflation

We now make contact with the KKLMMT model of warped brane inflation with only $\mathcal{N} = 1$ supersymmetry. This is done by embedding the previous $\mathcal{N} = 2$ system of D3- and D7-branes as a subsector of an orientifold with $\mathcal{N} = 1$ supersymmetry. This program can be performed within any $\mathcal{N} = 1$ toroidal orientifold based on $\mathbb{T}^6/\mathbb{Z}_{2N}$ or $\mathbb{T}^6/(\mathbb{Z}_{2N} \times \mathbb{Z}_{2M})$.[12] All of these models contain 32 D3-branes transverse to the compact space, plus at least one set of D7-branes wrapped on a 4-cycle, defined through an element of the orientifold group that acts as the reflection of four circles. In that case, the corrections to the gauge kinetic functions as discussed above for K3$\times\mathbb{T}^2$ can be literally copied to the $\mathcal{N} = 1$ model, only subject to more complicated projections on the Chan-Paton labels of the charged fields. Geometrically, this means that one has to impose the global symmetries of the orbifold space on the motion of the D3-branes, which was formerly unconstrained on $\mathbb{T}^2$. All other corrections to the gauge kinetic function of the D7-branes, if present, will be independent of the D3-brane scalars,

[11] According to the results of [26], this is the value at a minimum of the 3-form flux-induced potential at which $\mathcal{N} = 0$ or $\mathcal{N} = 1$ is preserved.

[12] Note, however, that in some cases the quadratic contribution in the inflaton field drops out of the gauge kinetic function due to the extra symmetry at the orbifold point. This does not happen in the $\mathbb{T}^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ model which is our main focus in the following.

and therefore of no concern here.[13] For the simple choice $\mathbb{T}^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$, the total modification compared to the results for K3$\times\mathbb{T}^2$ boils down to nothing but unimportant normalization factors, and a different gauge group. To adopt the notation of KKLMMT [12], we again locate all the D7-branes at the origin, and further concentrate on a single mobile D3-brane, denoting its position scalar by ϕ. This is the inflaton candidate.[14] We also rename S' to ρ as in the introduction. Explicitly, the gauge kinetic function of the D7-branes is then given by

$$f_7 = -i\rho - \frac{1}{4\pi^2} \ln \vartheta_1(\phi, U) + \cdots \ , \tag{27}$$

where the ellipsis indicates terms independent of ϕ. The modulus U is just one of the complex structure moduli of the background orientifold space, and assumed to be fixed through 3-form fluxes, i.e. U should be thought of as a function of the quantized flux parameters. When gaugino condensation takes place, one adds the superpotential

$$W_{\rm nonpert} \ \sim \ Ce^{-af_7} \ = \ w(\phi, U)e^{ia\rho} \ . \tag{28}$$

One can now extract the terms that potentially contribute to the inflaton mass by expanding $w(\phi, U)$ around some background value for ϕ. For $\phi = 1/2$, the function is even in ϕ, and one has

$$w(\phi + 1/2, U) \ \sim \ \vartheta_1(\phi + 1/2, U) \ = \ w_0(U) \left(1 + aw_2(U)\phi^2 + \cdots\right) \ , \tag{29}$$

where a factor a has been made explicit for later convenience. For general background values, there would also be a linear term.[15] In the language of KKLMMT (appendix F of [12]) this is a concrete identification of their parameter δ as a function of U, and thus as a function of the 3-form flux parameters, as argued above. One can now go back to the full scalar potential, insert the above superpotential, and find a correction to the slow-roll parameter η in the form [12]

$$\eta \ = \ \frac{2}{3}\left(1 - \frac{|\mathcal{V}_{\rm AdS}|}{\mathcal{V}_{\rm dS}}\Delta(U)\right) \ , \tag{30}$$

[13] Technically, the reason for this is that only the $\mathcal{N} = 2$ sectors of the orientifold depend on the moduli, and it is the special $\mathcal{N} = 2$ sector corresponding to the previously mentioned reflection along the four directions of the wrapped D7-branes that depends on the distance between the D3- and these D7-branes, i.e. the inflaton. The amplitudes arising in this sector are formally identical (up to some normalization factors) to the $\mathcal{N} = 2$ case discussed above, see App. B of [21].

[14] So this ϕ is one among the A_i of the previous section.

[15] The value $\phi = 0$ would not be a consistent choice in the present discussion, since then the D3-branes and D7-branes sit on top of each other, and new massless states appear. See also [34].

where $\mathcal{V}_{\rm AdS}$ is the value of the potential at the minimum before adding the tension of the anti-D3-branes, $\mathcal{V}_{\rm dS}$ is the value afterwards, and

$$\Delta(U) = -w_2(U) - 2(w_2(U))^2 \ . \tag{31}$$

Importantly, $\Delta(U)$ only depends on the quadratic coefficient $w_2(U)$ of $w(\phi, U)$, and not on the normalization $w_0(U)$, which is much harder to obtain. From (30) one easily infers that $\Delta(U)$ has to be positive in order to reach a lower value for η. It turns out that $\Delta(U) > 0$ for a wide range of values for U, cf. fig. 2.

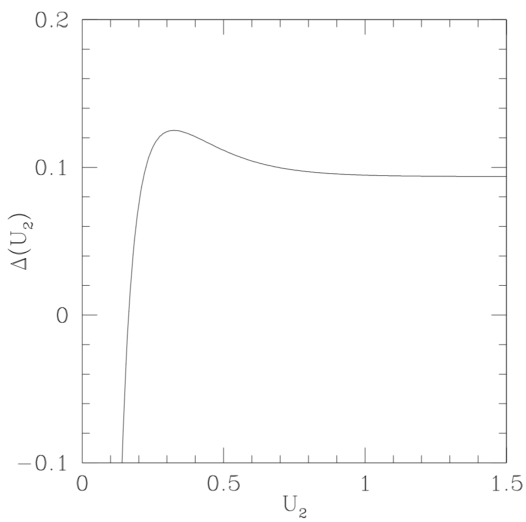

FIGURE 2: The function Δ of eq. (30); for positive values the inflaton mass is lowered by the one-loop corrections to the superpotential.

Thus, if the complex structure is stabilized in this range, η is lowered. Determining the actual value of η requires additional input, however; clearly our calculation does not capture details of the KKLMMT background, as our derivation of the one-loop correction to the gauge kinetic function does not take into account any effects of the warping or the background fluxes. Both can be expected to alter the result at least quantitatively, but qualitatively, we believe that our correction terms survive in the full background. Also, a conclusive answer to the question of how quantum corrections might help to solve the inflaton mass problem cannot be reached without also calculating the one-loop corrections to the Kähler potential. We will return to this problem in the near future [35]. Nevertheless, we consider it a merit of our result that it shows how terms quadratic in the inflaton field with moduli-dependent coefficients appear in

the superpotential in an explicit string theory model. This allows for moderate fine tuning of η by choosing flux parameters, as anticipated in [12].

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A Coupling correction and vertex operators

In this appendix we use standard vertex operator methods to verify our calculation in [21], that was performed in the background field formalism. The calculation presented here is not fully self-contained, and the interested reader should consult [21] and also [36] for more details on the conventions and notation. Concretely, we compute a 2-point function on an annulus with one end on the D7-branes and one on the D3-branes (cf. fig. 1). This is meant to demonstrate that the background field method does indeed give the correct answer for the one-loop correction to the D7-brane gauge coupling in the presence of dynamical D3-branes. We start with[16]

$$\langle V_A V_A \rangle_{\mathcal{A}_{37}} \sim \int_0^1 \frac{dq}{q} \int_0^q \frac{dz}{z} \int \frac{d^4p}{(2\pi)^4} \sum_{k=0,1} \mathrm{Tr}\Big[\theta^k V_A(\epsilon_1, p_1; z) V_A(\epsilon_2, p_2; 1) q^{L_0}\Big] \ , \quad (32)$$

where the zero-picture open string vertex operators are given by

$$V_A(\epsilon, p; z) = \lambda\, \epsilon_\mu (\partial X^\mu + i(p\cdot\psi)\psi^\mu) e^{ip\cdot X(z)} \ . \quad (33)$$

The second vertex operator has been fixed at $z_0 = 1$ and the Chan Paton matrix can be chosen, for instance, as

$$\lambda = \frac{1}{2}\,\mathrm{diag}(\underbrace{1, -1, 0, \,...,\, 0}_{16}, \underbrace{-1, 1, 0, \,...,\, 0}_{16}) \ . \quad (34)$$

[16] We are not interested in the overall normalization here and do not keep track of it in the following.

We use an off-shell prescription here, considering a small non-vanishing $\delta = p_1 \cdot p_2$.[17] Without such a prescription, all 2-point functions for massless external states are naively zero due to kinematics. The validity of this prescription can be checked by computing a 3-point function and taking a zero-momentum limit, or by curving the external space as an IR regulator, cf. [37].

Performing a calculation very similar to [36] but including the effect of moving the 3-branes away from the 7-branes along the torus (cf. [35]) yields

$$\langle V_A V_A \rangle_{\mathcal{A}_{37}} \sim [(p_1 \cdot p_2)(\epsilon_1 \cdot \epsilon_2) - (p_1 \cdot \epsilon_2)(p_2 \cdot \epsilon_1)] \sum_{k=0,1} \mathrm{tr}(\gamma_7^{-k} \lambda_1 \lambda_2) \times$$
$$\int_0^{i\infty} d\tau \sum_{\substack{\alpha,\beta \\ \text{even}}} \frac{1}{2} \eta_{\alpha,\beta} \, \frac{1}{4\pi^4 \tau^2} \frac{\vartheta[{}^{\alpha}_{\beta}]^2}{\eta^6} Z^{\alpha,\beta}_{\text{int, } k} \, \mathrm{tr}\left(\Gamma^{(2)}(\vec{a})\gamma_3^k\right) \int_0^{\tau} d\tilde{\nu} \, \left(\langle \psi(z)\psi(1)\rangle[{}^{\alpha}_{\beta}]\right)^2 ,$$

where $z = e^{2\pi i \tilde{\nu}}$. In contrast to [36] we immediately omitted the term from the contraction $\langle \partial X \partial X \rangle$ because we are only interested in the supersymmetric case, where its contribution vanishes after summing over spin structures. Finally, we set $\delta = 0$ everywhere except in the overall factor. This is admissible because in the case at hand the integration over $\tilde{\nu}$ does not lead to any poles in δ (cf. [27, 35] for more details). Furthermore, the internal partition function is given by [24, 38]

$$Z^{\alpha,\beta}_{\text{int, } k} = \frac{\vartheta[{}^{\alpha+1/2}_{\beta+k/2}](0)\vartheta[{}^{\alpha-1/2}_{\beta-k/2}](0)}{\vartheta[{}^{0}_{1/2+k/2}](0)\vartheta[{}^{0}_{1/2-k/2}](0)} , \tag{35}$$

the fermion correlator is [27]

$$\langle \psi(z)\psi(1)\rangle[{}^{\alpha}_{\beta}] = i\pi \, \frac{\vartheta[{}^{\alpha}_{\beta}](\tilde{\nu})\eta^3}{\vartheta_1(\tilde{\nu})\vartheta[{}^{\alpha}_{\beta}](0)} , \tag{36}$$

and $\Gamma^{(2)}$ is the sum over winding modes

$$\Gamma^{(2)}(\tau, \vec{a}) = \sum_{\vec{n}} e^{i\pi\tau(\vec{n}+\vec{a})^{\mathrm{T}} G(\vec{n}+\vec{a})} = \vartheta[{}^{\vec{a}}_{\vec{0}}](0, \tau G) , \tag{37}$$

with $\vartheta[{}^{\vec{a}}_{\vec{0}}]$ a genus-two theta function, and $\vec{a}$ the position of the 3-brane on the torus.[18] Now, using the identity

$$\sum_{\substack{\alpha,\beta \\ \text{even}}} \eta_{\alpha\beta}\vartheta[{}^{\alpha}_{\beta}](\tilde{\nu})^2 \vartheta[{}^{\alpha+h}_{\beta+g}](0)\vartheta[{}^{\alpha-h}_{\beta-g}](0) = \vartheta_1(\tilde{\nu})^2 \vartheta[{}^{1/2+h}_{1/2+g}](0)\vartheta[{}^{1/2-h}_{1/2-g}](0) \tag{38}$$

[17] Note that this is an entirely different δ than the one referred to above in the discussion of the KKLMMT model.

[18] More precisely, the position is $a^4\vec{e}_1 + a^5\vec{e}_2$ with $\vec{e}_1$ and $\vec{e}_2$ the basis vectors of the torus lattice. Moreover, in (37) we have set $\alpha' = 1/2$.

for $h = g = 1/2$ greatly simplifies the integrand as all the contributions of string oscillators drop out. After integration over $\tilde{\nu}$, we arrive at

$$\begin{aligned} \langle V_A V_A \rangle_{\mathcal{A}_{37}} \ \sim\ & [(p_1 \cdot p_2)(\epsilon_1 \cdot \epsilon_2) - (p_1 \cdot \epsilon_2)(p_2 \cdot \epsilon_1)] \\ & \times \sum_{k=0,1} \mathrm{tr}(\gamma_7^{-k} \lambda_1 \lambda_2) \int_{1/\Lambda^2}^{\infty} \frac{dt}{t} \mathrm{tr}\Big(\vartheta[\begin{smallmatrix}\vec{a}\\ \vec{0}\end{smallmatrix}](0, itG)\gamma_3^k\Big)\ . \end{aligned} \tag{39}$$

We changed the variable of integration to $t = -i\tau$, and introduced a UV cutoff Λ in the t-integral in (39). In complete analogy to [21], we can perform the traces and the integral to obtain

$$\begin{aligned} \langle V_A V_A \rangle_{\mathcal{A}_{37}} \ \sim\ & [(p_1 \cdot p_2)(\epsilon_1 \cdot \epsilon_2) - (p_1 \cdot \epsilon_2)(p_2 \cdot \epsilon_1)] \\ & \times \Big(\frac{\Lambda^2}{\sqrt{G}} - \ln \left| \frac{\vartheta_1(A, U)}{\eta(U)} \right|^2 + 2\pi U_2 (a^5)^2 \Big)\ , \end{aligned} \tag{40}$$

where we defined the complexified brane position

$$A = a^4 + U a^5\ . \tag{41}$$

The term proportional to Λ^2 in (40) is independent of the 3-brane scalars and drops out of the final result after adding the contribution of the 77-annulus and the 7-Möbius diagrams. Note, in particular, the difference of (41) to the T-dual variable used in [21], which will be denoted $\tilde{A} = U a_4 - a_5$ in the following. Let us compare (40) to the result of [21] by T-dualizing along both directions of the torus (actually along all six internal directions, but the K3 directions are not relevant here). The background-field result in the 59 picture, from (40) in [21], reads

$$\mathcal{A}_{95}^{\mathcal{F}^2} \ \sim\ \Big(\Lambda^2 \sqrt{G} - \ln \left| \frac{\vartheta_1(\tilde{A}, U)}{\eta(U)} \right|^2 + 2\pi U_2 a_4^2 \Big)\ . \tag{42}$$

If we now apply the rules of T-duality, i.e.

$$U \longrightarrow -\frac{1}{U} \quad , \quad \sqrt{G} \longrightarrow \frac{1}{\sqrt{G}} \quad , \quad a_i \longrightarrow a^i\ , \tag{43}$$

which imply

$$\tilde{A} \longrightarrow -\frac{1}{U} A\ , \tag{44}$$

we see that (42) is mapped according to

$$\begin{aligned} & \Big(\Lambda^2 \sqrt{G} - \ln \left| \frac{\vartheta_1(\tilde{A}, U)}{\eta(U)} \right|^2 + 2\pi U_2 a_4^2 \Big) \\ & \qquad \longrightarrow\ \Big(\frac{\Lambda^2}{\sqrt{G}} - \ln \left| \frac{\vartheta_1(-A/U, -1/U)}{\eta(-1/U)} \right|^2 + 2\pi \frac{U_2}{|U|^2} (a^4)^2 \Big)\ . \end{aligned} \tag{45}$$

Using standard modular transformations (cf. [37]), one easily verifies that (45) coincides with the bracket in (40).

As (40) gives the dependence of the one-loop correction to the 7-brane gauge coupling on the 3-brane scalars, this confirms the results derived in the T-dual picture using the background field method in [21].

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